Abstract of Dissertation

„Classification of Netinvariant Collineations and Application of an Elliptic Net in Triangle Geometry“

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In the first part of this dissertation is given a classification of all those collineations in real projective space, that leave invariant a certain elliptic linear congruence. The start point of this classification is the set of all Möbius transformations of the real Möbius plane, because the elliptic congruence is a model of this plane embedded in projective space and because each netinvariant collineation is available as an up in space „lifted“ Möbius transformation.

In the second part of this work the main facts of elliptic netprojection are described. Lines in projective space are mapped to Möbius circles via the rays of the elliptic congruence. The netprojection leaves invariant the cross ratio. Geometric relationships in the Möbius plane can be described by corresponding relations in projective space. An example is given by Miquel's configuration of circles.

In the last and main part the elliptic net and the netprojection are used to show some facts of triangle geometry via geometric elements of projective space. Certain parabolas belonging to a triangle step on stage (Artzt parabolas), further the symmedian and the two Brocard points and also the Brocard angle. It can clearly be seen by the used space methods how all these facts are closely connected to each other. This connection is given by a space line belonging to the triangle which is named here as Brocard line.

Possibly many more phenomena of triangle geometry can be easier made understandable by the use of an elliptic linear congruence as if only using methods of plane geometry. In that direction this dissertation wants to give a motivation.